An inverse kinematics approach to hexapod design and control

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An inverse kinematics approach to hexapod design and control

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ABSTRACT

Inverse kinematics can be used to determine the interaction between the motions of the individual linear actuators and the motion of the moveable platform of a hexapod. This paper presents concepts that can be utilized by the designer or user to determine fundamentals such as resolution, motion limits, actuator loading, and stiffness of a given hexapod design. Hexapod platforms have found use in high-end systems when precision positioning and multiple degrees of freedom are required in optical, industrial, or robotic applications. Hexapods make use of parallel kinematics to achieve high levels of precision and accuracy and can often outperform traditional methods of achieving motions involving multiple degrees of freedom. Traditional methods may involve serial kinematics in the form of stacked translation and rotation stages. They have the advantage of being conceptually simple and straightforward to implement, but often suffer from decreased stability. Despite the advantages of stability and the freedom of motion hexapods offer, hexapods are often avoided because of their non-intuitive nature. We endeavor to present a straight-forward approach to understanding hexapod movements and provide insight into the advantages and limitations of hexapod platforms.

Keywords: optomechanical, hexapod, kinematics, motion-control, positioning

1. INTRODUCTION

Hexapods, also known as Stuart platforms, have found broad application since their introduction over 50 years ago.[1] These applications include flight simulators, medical instruments, industrial machine tools and precision metrology. Hexapods are parallel kinematic structures capable of controlled motion in six degrees of freedom. Generically they consist of a stationary base and a mobile platform connected by six struts (legs) of adjustable length. These can be thought of to be acting in parallel on the mobile platform and all six are required to constrain it fully.

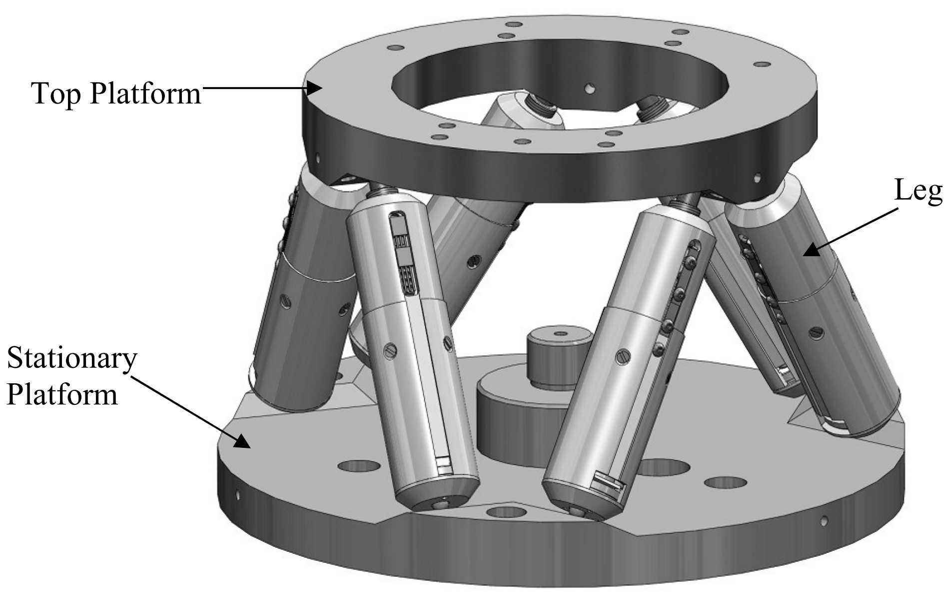

Fig. 1. 6-6 Hexapod Platform

The general "6-6 platform" as depicted in Fig. 1 is a hexapod consisting of six joints on the stationary platform, and six joints on the mobile platform. The mathematical problem of finding each leg length for a desired position of the mobile platform is one of inverse kinematics that can be solved in closed form. However, the problem of determining the position and orientation of the platform for six known leg lengths is one of forward kinematics, which has no known closed form solution[2]. Because the inverse kinematics problem is readily solved, we will show how aspects of the solution may be usefully applied in both the design and control of a hexapod platform.

2. THE INVERSE KINEMATICS PROBLEM

2.1 Problem Statement

The inverse kinematics problem is that of determining the length of each leg knowing the position of the moveable platform relative to the stationary base. This is mathematically straightforward to solve. Consider the geometry of a hexapod shown in Figure 2.

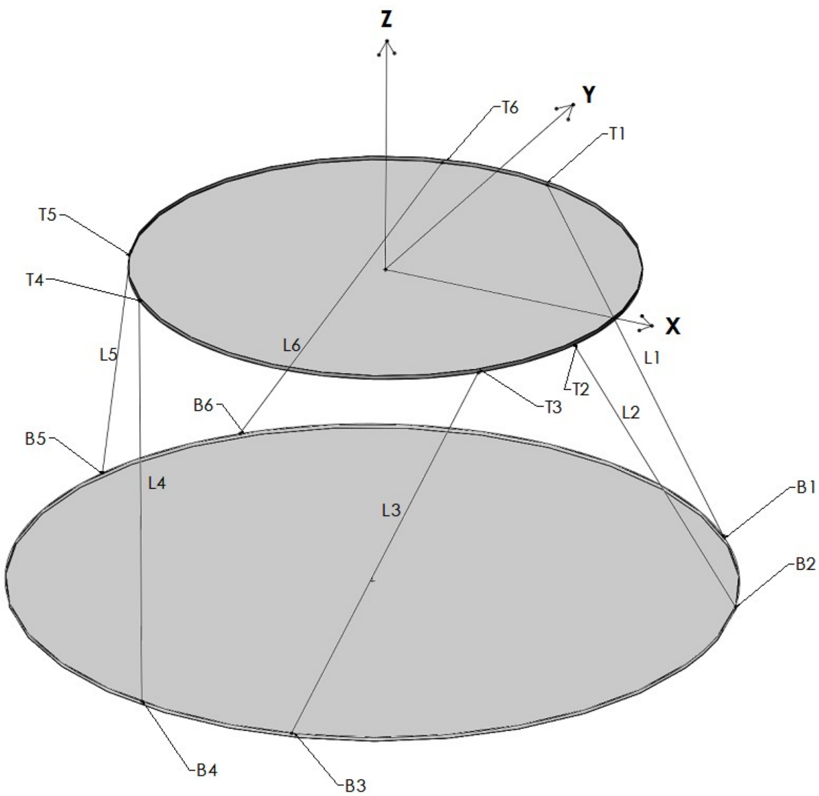

Figure 2

Here,

$$L_i = \sqrt{(T_{ix} - B_{ix})^2 + (T_{iy} - B_{iy})^2 + (T_{iz} - B_{iz})^2} \qquad \text{Eq. 1}$$

Desired translations and rotations of the top platform can be directly applied to the coordinates at points $T_{1\text{-}6}$. The post-movement leg lengths, L_i, can then be calculated for the new top platform joint coordinates, and then compared with the pre-movement leg lengths, to determine the required change in the length of each individual leg.

2.2 Limitations

Inverse kinematics yields leg lengths for known platform positions. However, if one desires to move the motion platform from an initial position to a desired position; simply commanding each leg to change in length by a known amount may cause an undesirable trajectory to be followed.[2] Figure 2 depicts a hexapod geometry where legs 1 and 6 lie within a plane parallel to the X-Z plane (see Figure 3), when the motion platform is centered over the stationary platform. With this geometry, a translation of the top platform in the positive or negative Y direction without a change in elevation will require legs 1 and 6 to lengthen. Therefore, if the top platform is required to translate from a position in negative Y space to a position in positive Y space, legs 1 and 6 would initially need to get shorter and subsequently get longer in order to maintain a constant elevation throughout the trajectory.

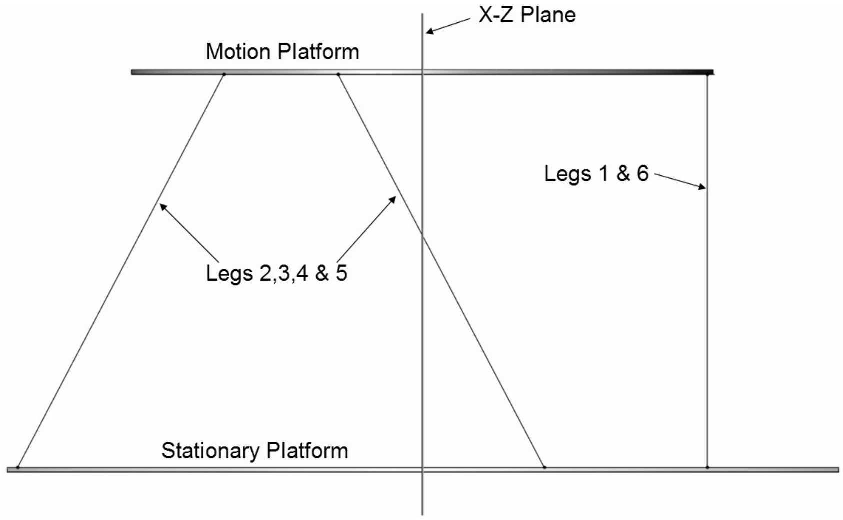

Figure 3: Schematic side view of hexapod

If "Inverse Kinematics" finds the leg lengths for a known motion platform position, then "Forward Kinematics" finds the position of the motion platform for known leg lengths. Unfortunately, inverse kinematics is unable to uniquely determine the motion platform's position from a known set of leg lengths.

Consider for example, if in Figure 2 all legs are of equal length, an equivalent mathematical solution would place the motion platform directly underneath the stationary platform. This is of course physically impossible for the hexapod to achieve, despite the validity of the mathematical solution to the forward kinematics problem.

Conceptually, we may consider each leg to define a sphere, centered on the joint at the stationary platform, whose radius is equal to the leg length. The forward kinematics problem, then, is to find all the positions where a field of six points with a fixed relationship to each other can lie coincident with the six spheres representing the legs.

There have been many techniques published (e.g. Schipani[3] (2006), Korobeynikov(2005)[4], Jakobovic et al.(2002)[5]) that address solving the forward kinematics problem; in general, these techniques require iterative algorithms and established boundary conditions to avoid arrival at solutions that are legitimate mathematically, but physically impossible. In Section 4.2, we describe how we make use of modern CAD software solve the forward kinematics problem and test the inverse kinematics algorithm.

3. APPLICATION OF INVERSE KINEMATICS TO HEXAPOD DESIGN

Hexapod design can be divided into four sub-categories: hexapod geometry, leg (actuator) design, joint design, and motion-control. Establishing an inverse kinematic model of the hexapod is a useful tool for addressing each of these steps. In the simplest form, the model consists of the following algorithm:

- input the initial conditions of the hexapod, as dictated by the chosen geometry
- calculate the leg lengths using these initial conditions
- input a secondary location of the motion platform
- re-calculate the leg lengths plus related or derived parameters.

The initial conditions are defined by the locations of the joints in the stationary platform and in the motion platform. In most cases, we assume that for the initial conditions, the motion platform will be centered within its physical working envelope. With the inverse kinematic model established, the relationship between the leg lengths and the motion platform can be determined and subsequent calculations giving further information regarding the state of the hexapod assembly can be performed.

3.1 Resolution

The range of motion of a hexapod will most likely not be equal in the X, Y, and Z directions, and the resolution may differ in the three axes of translation and rotation. Given that simple linear moves and/or axial rotations may require coordinated changes of the leg lengths of varying amounts, a direct relationship between the resolution of a single leg's linear actuator and the motion platform is difficult. However, once a model is established, it becomes straightforward to determine the linear movements required by each actuator to achieve a coordinate translation of the motion platform. These required movements can then be compared to the resolution of the linear actuators to determine whether the desired motion resolution can be achieved. Then, for a desired minimum resolution of the motion platform, the distance required for each actuator to move can be calculated.

Figure 2 shows an example 6-6 hexapod where each of the legs is nominally 100mm long. An inverse kinematic model was created and the required changes to the leg lengths were calculated to move the upper platform 0.1mm in the X, Y, or Z directions. The results of this analysis are tabulated in Table 1.

	CHANGES IN LEG LENGTH (mm)		
	0.1mm X	0.1mm Y	0.1mm Z
LEG 1	-0.0518	0.0001	0.0855
LEG 2	-0.0259	-0.0449	0.0855
LEG 3	0.0260	0.0449	0.0855
LEG 4	-0.0259	0.0449	0.0855
LEG 5	0.0260	-0.0449	0.0855
LEG 6	0.0519	0.0001	0.0855

Table 1: Changes in leg lengths required for 0.1mm translation of the motion platform

A minimum resolution of 0.025mm would seem to be required to set the lengths of the legs for an X direction movement. In the Y direction, there are two legs that are virtually unchanged; these are the two legs that lie in the X-Z plane. Therefore, near their nominal position, they act primarily as a parallel linkage. Discounting these two legs, the other four actuators would need to move 0.045mm. The Z direction is the most straightforward, as all actuators move an identical amount of 0.085mm. It is evident that a direct link between the desired resolution of the motion platform (0.1 mm) and a required individual leg resolution is difficult.

3.2 Motion Limits

Multiple factors can affect the motion limits of the moveable platform. The factors include the stroke of the linear actuators, the allowable angles of the leg joints, and any possible interference between the legs and/or the platform[5]. Figure 4 depicts the motion envelope boundary of a hexapod in the X, Z directions at $Y = 0$, while Figure 5 depicts the motion envelope boundary of a hexapod in the X, Y directions at an elevation of $Z = 0$ with all actuators having an identical maximum stroke.

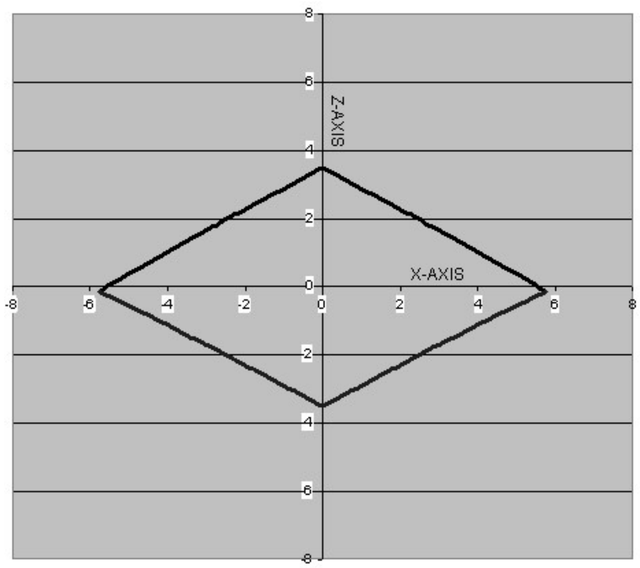

Figure 4: Motion limits at Y = 0

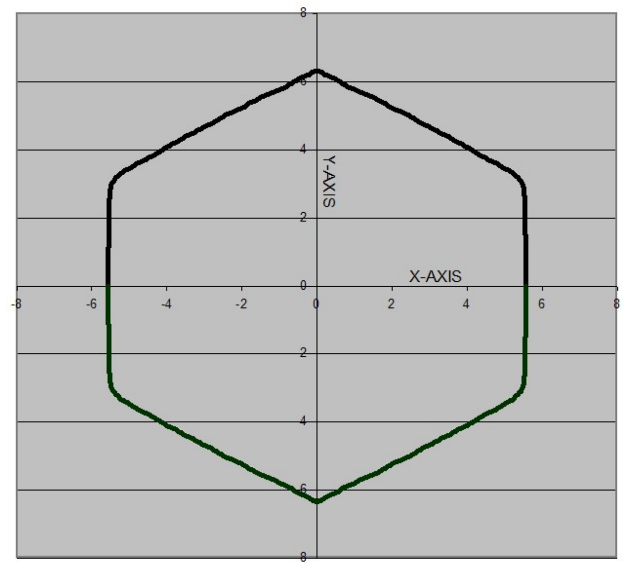

Figure 5: Motion limits at Z=0

The symmetric hexagonal shape of the motion envelope depicted in Figure 4 is a direct result of the hexapod geometry. This can be contrasted against the rectangular working envelope of a stacked linear stage arrangement to achieve motion in the X-Y plane.

The allowable leg joint angles will be determined by the type of joint chosen. Multiple leg joint designs are commonly used which may include spherical bearings, universal joints, or flexures. Each type will have a maximum operable angle. The maximum angle may represent a physical limit, or a limit at which performance will be degraded. Assuming that when the motion platform is in its nominal position the joints are at an angle of 0°, the joint angle for each leg at each new platform position may be calculated utilizing the law of cosines, as expressed in Figure 6 and Equation 2:

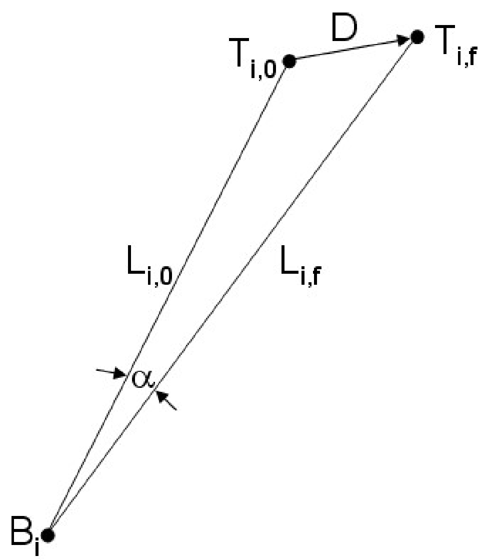

Figure 6: Calculation of joint angles for a final position relative to an initial position

$$\alpha = \cos^{-1}(L_{i,f}^2 + L_{i,0}^2 - D^2) \qquad \text{Eq. 2}$$

where

$$D = \sqrt{(T_{ix,f} - T_{ix,0})^2 + (T_{iy,f} - T_{iy,0})^2 + (T_{iz,f} - T_{iz,0})^2} \qquad \text{Eq. 3}$$

Calculating interference requires identifying the candidate items that may interfere. In the case of leg-to-leg interference, a minimum distance between each pair of adjacent legs can be determined and then compared with the known radii of the legs at the correct axial distance along the leg length.

3.3 Leg loading and stiffness

In an ideal hexapod, each leg acts as a strut that carries tensile or compressive loads only. The loading of each leg will be dependent on the payload position, the position of the motion platform relative to the stationary platform, and the orientation of the system relative to gravity. In the nominal position of the hexapod shown in Figure 2, each leg forms an identical angle relative to a horizontal plane. Thus, each leg will have an identical vertical load component when the payload or self-weight of the motion platform is centered.

When each leg is treated as an ideal strut, the horizontal and vertical stiffness will be a function of the spring constant of each leg. The spring constant of each leg will be a function of the total axial stiffness of the linear actuator, the joint ends, and any other mechanisms involved. The spring constants of each of these may be summed as in Equation 4.

$$1/K_1 + 1/K_2 + 1/K_3 = 1/K_T \qquad \text{Eq. 4}$$

For any position of the upper platform relative to the lower platform, the ability of the legs to resist a horizontal or vertical load can be found by converting the axial stiffness of each leg into X, Y, and Z components. In this way the stiffness in X, Y, Z, θX, θY, and θZ is found.

4. APPLICATION OF INVERSE KINEMATICS TO HEXAPOD CONTROL

4.1 Pseudo Trajectory Control

Some level of trajectory control can be achieved by dividing any movement of the motion platform into a series of smaller moves. In this manner, a linear trajectory from a start position to an end position may be approximated by minimizing the deviation of the platform from the straight trajectory. Alternatively, if the motion platform needs to avoid a stationary object, or some other constraint is applied, a path other than a straight line can be approximated.

4.2 Implementation Example

In the following example, a control GUI (Graphical User Interface) for a hexapod is implemented using Microsoft Excel®. The initial conditions are entered as twelve joint locations (six upper and six lower) in space. Using a combination of sliders, macros, and user entry cells, platform motion commands in terms of X, Y, Z, θX, θY, and θZ are entered. The desired motions are then utilized to define the new positions for the upper six joints and from these the leg lengths, joint angles, and leg loadings are computed. In addition, both the position of the upper platform within the working envelope and the position of the actuators relative to their total achievable travel are calculated.

We test the control using the "Design Table" functionality within SolidWorks®. SolidWorks is advanced CAD (Computer Aided Design) software that allows fully parametric models and sketches to be created, so that when any parameter is adjusted, the model will update all linked parameters accordingly. With "Design Tables," when a parametric sketch or sketches are linked to an Excel workbook, the output of any cell can drive a dimension within the SolidWorks sketch. Figure 7 shows the GUI created to "drive" the model hexapod.

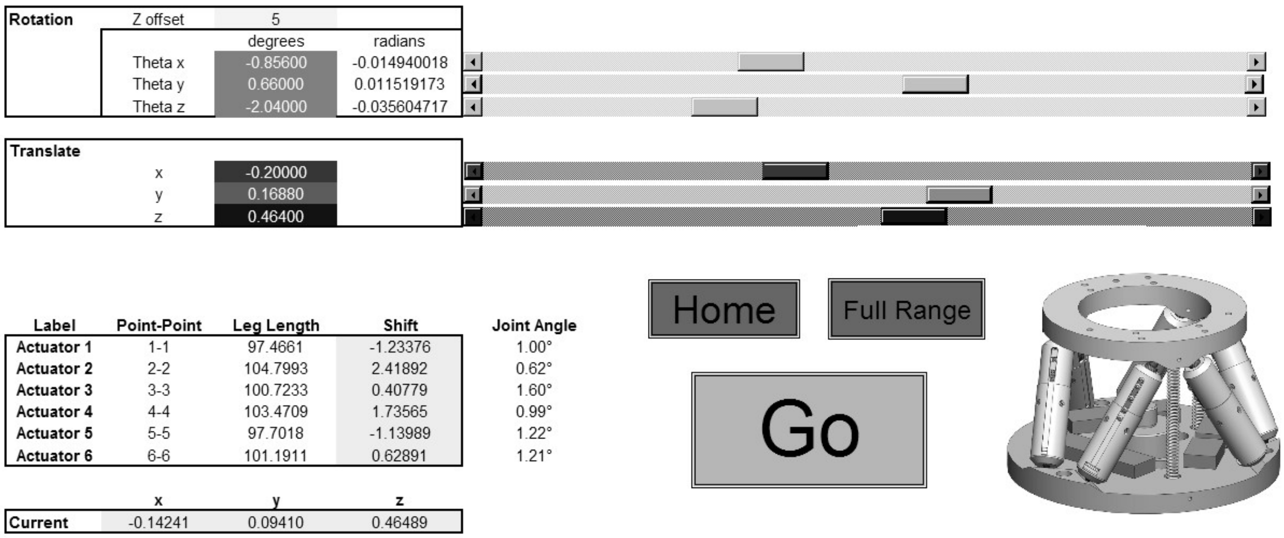

Figure 7: Example of Excel based hexapod control GUI

Because Solid works (as well as other modern CAD packages) is able to solve the forward kinematics problem, we can independently verify the desired end position of the motion platform using the leg lengths calculated by the spreadsheet. The leg lengths calculated in the spreadsheet drive the dimensions of the 3d CAD sketch shown in Figure 8, while all other dimensions defining the geometry remain fixed. As the spreadsheet is updated, new leg lengths are calculated and input into the 3d sketch. Because only the legs lengths are varying, the circle representing the motion platform moves as a rigid body. In this way, the functions within the spreadsheet can be tested and debugged through this form of virtual prototyping, before hardware is ever utilized.

Realizing that multiple solutions may exist for a given set of leg lengths; it is possible that SolidWorks will either fail to move to the expected location or fail to find a solution at all. However, in our experience, as long as the desired solution is within the working envelope of the physical implementation, SolidWorks will converge on the correct solution.

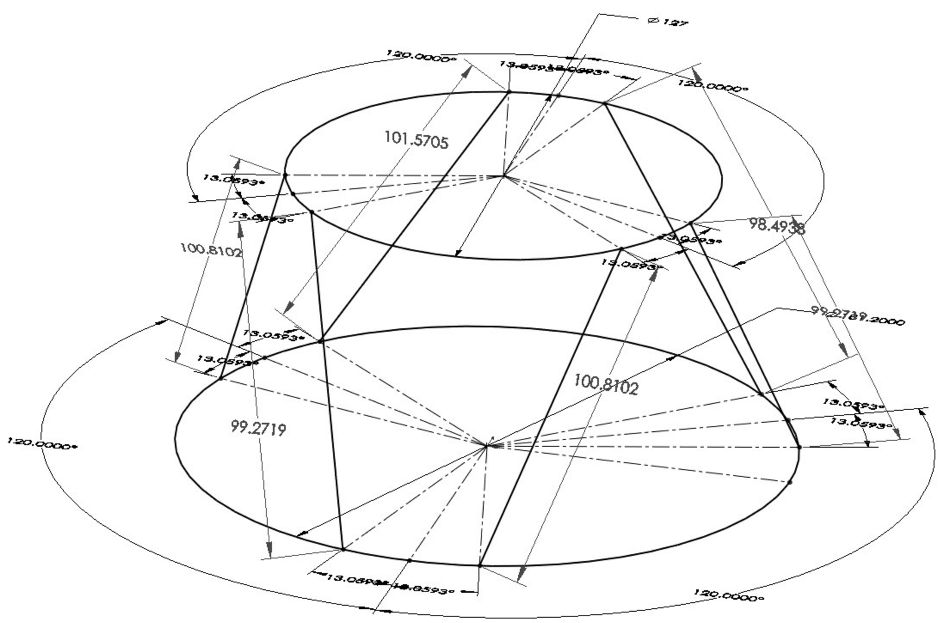

Figure 8: SolidWorks “3dSketch” used to simulate hexapod movement

5. ADVANTAGES AND LIMITATIONS OF HEXAPODS

5.1 Advantages

A hexapod is a form of parallel manipulator with many advantages compared to stacked serial manipulators. In its ideal form the hexapod is an example of an exact constraint device. This means that there are exactly enough constraints to define the motion platform, and no more. Exact constraint devices are highly desirable when precision motion is required because they are, by their nature, more predictable and repeatable.

Additionally, a hexapod is perhaps the most efficient method of achieving six degrees of freedom in terms of space, economics, and possibly design or implementation time. A traditional method might consist of three linear stages, a tip/tilt stage, and a rotary stage, stacked on top of each other. Each axis will require an actuator and the mechanism to define the desired degree of freedom. Each of these mechanisms may have different drive requirements and different motion characteristics that need to be dealt with. By comparison, a hexapod likely has six identical linear actuators with identical drive parameters to accomplish the same six degrees of freedom. With its "exact constraint design," a hexapod can achieve high levels of stability and precision with less physical mass and size than the stacked serial manipulators. This simplicity, achieved through component commonality, leads to more economical design implementations.

5.2 Limitations

While hexapods can offer an elegant solution for providing six degrees of freedom, they clearly have characteristics that can limit there usefulness. The primary limitation is the fact that X, Y, Z, θX, θY, and θZ moves are not inherently independent from one another. With serial manipulators, each actuator will affect a change in only one degree of freedom, whereas with a hexapod, a change of any one linear actuator causes changes in multiple degrees of freedom. Most of the other foreseen limitations of hexapods stem from this interdependency. For example, it is virtually impossible to manipulate a hexapod manually (by use of a joystick or other "jogging" control) to achieve a useful desired motion. If precision is required not only in achieving specific locations of the motion platform, but also in the path taken between these locations, sophisticated controls must be implemented to keep the motion of the moveable platform within acceptable error limits.

Hexapods may also not be the best choice if the required resolution, or strokes of the axis of motion, vary greatly. For instance, if a device requires very large, course movements in the X and Y directions, but very precise short strokes in the Z direction, then a combination of serial manipulators may be a better choice.[6]

6. SUMMARY

We have used inverse kinematics to create a mathematical model that correlates the motion of a hexapod's moveable platform with the individual motions of the linear actuators within the legs. The inverse kinematics model can also be used as a simple algorithm to control a hexapod with certain limitations, specifically in regards to trajectory control and accounting for deviations of the actuators from their intended positions. A straightforward, spreadsheet based control software has been described and has been integrated with SolidWorks, providing a virtual prototype of a hexapod to be used for checking both the design of the physical unit, as well as its control algorithms.

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